Water and sediment supply conspire to determine channel size, shape, composition, & ecological function.

Sediment & Sediment Transport
Peter Wilcock 11 February 2008

There are two basic transport calculations

- Incipient motion. Will grains move? Threshold channel: flow is no larger than the ‘critical discharge’ $Q_c$ that produces grain motion.
- Transport Rate. What is the transport rate $Q_s$? Mobile-bed channels transport supplied sediment.

These are different problems!!! $Q \geq Q_c$ does not mean that the sediment supplied can be transported!

Consider a highway toll booth ...
The first transport problem: incipient grain motion

What drives grain motion?

\[
\frac{\text{Flow force on bed}}{\text{bed area}}: \tau_o \quad \text{The Shields Number}
\]

\[
\text{(Grain weight): } (s-1)\rho g \frac{\pi}{6} D^3
\]

Number of grains/area \( \propto 1/D^2 \)

Shields Number: \( \frac{\text{Flow Force}}{\text{Grain weight}} = \frac{\tau_o}{(s-1)\rho g D} \equiv \tau^* \)

This is THE most important variable in sediment transport

\( D \) grain size; \( g \) acceleration of gravity; \( \rho, \rho_s \) water, sediment density; \( s = \rho_s / \rho \)

The second transport problem: transport rate

Most sediment transport problems are defined in terms of total sediment transport rate \( Q_s \) and water discharge \( Q \).

A relation giving \( Q_s \) as a function of \( Q \) is a sediment rating curve.

\[
Q_s = aQ^b \quad (1)
\]

\( Q_s \) tons per day (kg/hr, Mg/day) and \( Q \) ft\(^3\)/s, or cfs (m\(^3\)/s, or cms)

Example: with a sediment rating curve such as (1) and a record of discharge (e.g. the daily mean value of \( Q \) for 25 years), you can calculate the total sediment load (the sediment yield) by using (1) to calculate the tons of sediment transported for each day and adding up all 9131 or so values to get a total sediment yield for 25 years.

Warning: Sediment Rating Curves can shift &

May need a much shorter time interval
The obvious thing to do: measure $Q$ and $Q_s$ and directly develop a sediment rating curve from the data.

Collect $Q_s$ at different $Q$, plot on log-log paper, a straight line follows the relation

$$\log(Q_s) = \log(a) + b \log(Q)$$

Exponentiate, and you have

$$Q_s = a Q^b$$

$$Q_s = 1.4 \times 10^{-25} Q^{5.9}$$

Now to modeling transport rates. If a sediment rating curve is what we use in application, isn’t there a general one available?

Will 1,000 cfs in Minebank Run move the same amount of sediment as 1,000 cfs in the Susquehanna River?

One useful trick: $Q_s = a Q^b$  \[ \frac{Q_s}{Q_{sr}} = \left( \frac{Q}{Q_r} \right)^b \]

It’s nice that $a$ cancelled, but for this model to be general, the rate of change of $Q_s$ with $Q$ (i.e. the exponent $b$) must be the same everywhere. But we will find that $b$ varies from little more than one to more than ten!.

Basically, we are looking for a flow variable that can be accurately scaled, such that we have a general model. For this we use the bed shear stress $\tau$. 

Selway River

Transport Rate (tons/day)

Discharge (cfs)

$>4 \text{mm}$

$4 \text{mm}$

C1

Bridge

C2

Loglinear

$1$, $6$, $25$, $5.91.4$, $10$

$<4 \text{mm}$
(Flow force on bed)/(bed area): $\tau_0$  

(Grain weight): $(s-1)\rho g \frac{\pi}{6} D^3$

Number of grains/area $\propto 1/D^2$

Shields Number: $\frac{\text{Flow Force}}{\text{Grain weight}} = \frac{\tau_0}{(s-1)\rho g D} \equiv \tau^*$

This is THE most important variable in sediment transport

The Einstein Transport Parameter

Volumetric transport rate/width $q_s$

Sediment fall velocity $w \propto \sqrt{(s-1)gD}$

$\tau^* = \frac{\text{transport rate}}{\text{fall velocity} \times \text{grain size}} = \frac{q_s}{\sqrt{(s-1)gD^3}}$

A typical transport model: $q^* = a(\tau^* - \tau_c^*)^b$

Meyer-Peter & Müller: $q^* = 8(\tau^* - \tau_c^*)^{3/2}$

\[
\frac{q_s}{\sqrt{(s-1)gD^3}} = 8 \left( \frac{\tau}{(s-1)\rho g D} - \frac{\tau_c}{(s-1)\rho g D} \right)^{3/2}
\]

\[
q_s = \frac{8}{(s-1)g \rho^{3/2}} (\tau - \tau_c)^{3/2}
\]

$q_s = 78.7 (\tau - \tau_c)^{3/2}$

for $q_s$ in (tons/day)

and $\tau, \tau_c$ in (Pa)

“simpleMPM.xls”
A typical transport model: \( q^* = a(\tau^* - \tau_c^*)^b \)

Meyer-Peter & Müller: \( q^* = 8(\tau^* - \tau_c^*)^{3/2} \)

One more transport variable, \( W^* \)

\[ W^* = \frac{q^*}{(\tau^*)^{3/2}} = \frac{(s - 1)gq_s}{(\tau^*/\rho)^{3/2}} \]

One more stress, the reference stress \( \tau_r \)

Convert the M-PM formula to \( W^* \) and incorporate a reference shear stress. First, we divide M-PM by \((\tau^*)^{3/2}\) to get

At \( \tau^* = \tau_r^* \), \( W^* = W^*_r = 0.002 \). Dividing by 8, raising both sides to the 2/3 power produces

\[ 0.004 = 1 - \frac{\tau_c^*}{\tau_r^*} \]

\( \tau_c^* = 0.996\tau_r^* \)

Meyer-Peter & Müller: \( W^* = 8\left(1 - \frac{\tau_c^*}{\tau_r^*}\right)^{3/2} \)

Reference shear stress? Critical shear stress?

![Graphs showing reference and critical shear stress](image)
How the reference shear stress gets used

The Difference Between $\tau_c$ and $\tau_r$

$\tau_c$: boundary between motion & no motion. Definable exactly for an individual grain; definable statistically for a river bed

Hard to measure

$\tau_r$: the stress associated with a small, agreed-upon transport rate ($W^* = 0.002$) provides a threshold for a transport relation

Easy to determine from transport observations
The Measurement of $\tau_c$ and $\tau_r$

$\tau_r$ : from transport observations

$\tau_c$ : tracers (painted rocks, magnetic rocks, radio rocks, rock scum) answer the question “did the grain move at all?”

(place tracers, return after flow, measure # moved, repeat for range of flows, develop relation between %entrained, grain size, and flow)

Need large numbers of grains for reliable sample
Need to place “naturally”

The Application of $\tau_c$ and $\tau_r$

$\tau_r$ : its purpose is applied: a threshold for a transport relation

It provides a measurable surrogate for $\tau_c$

$\tau_c$ : sometimes we are interested in the entrainment of individual grains (or the proportion of grains entrained). For example,

“At what discharge do 90% of the surface grains become entrained, thereby providing access to the subsurface and some flushing action?” Or,

“At what discharge will 1% of the surface grains be entrained, thereby indicating that our threshold channel is beginning to fall apart?”
Recap

- Channel change driven by changes in the Balance between water supply, sediment supply, & streams
- Other factors (esp. vegetation) can dominate
- Sediment: huge range of sizes, usually resting
- Sediment moves as bed load and suspended load, but practical problems better defined in terms of bed material load and wash load
- Our fundamental goal in estimating transport is some form of sediment rating curve \( Q_s = f(Q) \)
- One approach is to directly measure transport
- Transport predictions require a general transport model, which requires a scalable flow variable, for which we use bed shear stress \( \tau \)
- Meyer-Peter & Muller is a representative model note that it is highly nonlinear ……
- There are TWO Transport Problems; this is why we have a critical and a reference shear stress
Will sediment move? Is \( \tau_o > \tau_c \)?

As \( Q \) increases, up go stage & \( R \) (controlled by resistance and continuity).

For a given \( S \), we can find boundary stress using momentum \( \tau_o = \rho g R S \) and compare to \( \tau_c \).

At what \( Q \) does \( \tau_o = \tau_c \)?

Combining \( Q = UA \) and \( U = \frac{\sqrt{S}}{n} R^{2/3} \): \( Q = \frac{\sqrt{S}}{n} AR^{2/3} \)

If we know \( n \) and \( \tau_c \), we can find \( R_c \) from \( \tau_c = \rho g R_c S \)

and then calculate \( Q_c \). Or, we can simply adjust \( Q \) until \( R = R_c \).

X/S.xls illustrates the solution for \( Q_c \)

For the stage, flow depth, and cross section you have specified, use Manning's eqn to find the discharge \( Q \) for a value of the roughness in cell E4.

You can also calculate the boundary stress \( \tau_o \) and compare it to the critical stress \( \tau_c \) needed to entrain grains of size \( D \) specified in cell D10 (given a value of critical Shields Number given in cell D11).

For the slope, flow depth, and cross section you have specified, use Manning's eqn to find the roughness \( n \) for a value of the discharge \( Q \) in cell E15.

No cutting and pasting! No inserting or deleting rows!
Sediment Transport Why its hard to estimate 0

Example calculation using Meyer-Peter and Muller for a channel with slope $S = 0.002$, roughness $n = 0.025$, and width $b = 15m$. The solid curve uses $r_c = 0.045$ and $D = 45$ mm. The dashed curve uses $r_c = 0.045$ and $D = 30$ mm.

Note that at discharge $Q = 55$ cms, one curve indicates zero transport and the other a transport rate of 80,000 kg/hr.
Sediment Transport *Why its hard to estimate I*

Transport rate is a *steep, nonlinear* function of bed shear stress.

Small error in $\tau$ can produce big error in $q_s$.

Three things make it difficult to accurately estimate $\tau$:

1. Accelerations in *unsteady, nonuniform* (i.e. real) flow
2. Only a portion of the total $\tau$ acts to transport sediment
3. $\tau$ varies locally. Because transport is a nonlinear fn of $\tau$, estimates based on total $\tau$ will be in error

\[ \tau_0 = \rho g R \left( S - \frac{\partial h}{\partial x} - \frac{U}{g} \frac{\partial U}{\partial x} - \frac{1}{g} \frac{\partial U}{\partial t} \right) \]

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\[ \tau_0 = \rho g R \left( S - \frac{\partial h}{\partial x} - \frac{U}{g} \frac{\partial U}{\partial x} \right) \]

\[ \tau_0 = \rho g R_f \]

**NOTE:**

Backwater programs compute $S_f$. 
The Drag Partition

So far, the stress we are talking about is the total stress $\tau_0$, or force per area, that the flow exerts on the boundary of the channel. But only a portion of the total $\tau_0$ acts on the sediment grains to produce transport. To estimate transport rates, we need to partition $\tau_0$ into the part that acts only on the sediment grains. We call this the skin friction, or grain stress $\tau'$. There is no direct way to do this. We will develop an approximate method, using Manning’s equation.

Three things make it difficult to accurately estimate $\tau$:  
1. Accelerations in unsteady, nonuniform (i.e. real) flow  
2. Only a portion of the total $\tau$ acts to transport sediment  
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Note that there is a ‘depth-slope’ product in the Manning eqn:

$$U = \frac{\sqrt{SR^{2/3}}}{n}$$

$$(\rho g)^{2/3} S^{1/6} n U = (\rho g RS)^{2/3}$$

$$\rho g S^{1/4} (n U)^{3/2} = \tau_0$$

It’s just Manning’s eqn, solved for shear stress. We can estimate the part of the roughness due to the bed grains:

$$n_D = 0.013 D^{1/6} \quad \rightarrow \quad \rho g (0.013)^{3/2} (SD)^{1/4} U^{3/2} = \tau'$$

$$\tau' = 17 (SD_{65})^{1/4} U^{3/2}$$

where we use $D = 2D_{65}$

$\rho = 1000 \text{ kg/m}^3$ & $g = 9.81 \text{ m/s}^2$
What does this mean? Part of the roughness is due to the grains and part is due to other stuff (trees, bends, flow expansions, etc.).

If we use the Manning eqn to estimate the boundary stress $\tau$, and use the Manning-Strickler grain roughness, we have an estimate of the portion of the total stress acting on the grains – the grain stress.

$$\tau' = 17 (SD_{65})^{1/4} U^{3/2}$$

The grain stress $\tau'$ depends mostly on $U$ and less on $S$ and $D$ – a good thing.

These seemingly obscure bit of hydraulics will return and save our bacon when we calibrate a transport formula.

Basically the same drag partition suggested by Meyer-Peter & Muller in the ‘40s. Using Manning’s $n$ incorporates general experience.

$$\tau_o = \rho g S^{1/4} (nU)^{3/2}$$

$$\tau' = \rho g S^{1/4} (nDU)^{3/2}$$

$$\frac{\tau'}{\tau_o} = \left(\frac{n_D}{n}\right)^{3/2}$$

$$\tau' = \left(\frac{n_D}{n}\right)^{3/2} \tau_o$$
Sediment Transport *Why its hard to estimate II*

What is $D$ for a reach?

What if $D$ is not even in the reach?

Critical shear stress depends linearly on grain size, but the grain size of the transport is poorly known!

Transport rate is a steep, nonlinear function of bed shear stress

Small error in $\tau_c$ can produce big error in $q_s$

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Estimating Sediment Transport: Why its hard

- Sediment transport is a local process: between individual grains and the near-bed flow they encounter
- The transport process is highly nonlinear: a very steep function with a threshold between motion and no motion
- Real streams have considerable variability in topography, flow, and bed composition
- The local flow acting on grains is hard to estimate
- The distribution of bed grain sizes is hard to determine
- You never know either local flow or local bed composition in detail, so you have to estimate
- Errors in estimated transport rates are typically very large (often well in excess of an order of magnitude!)
- Next, we examine an approach to provide reliable transport estimates, of greater accuracy, with acceptable effort
• If a good transport estimate is required:  
  field observations are needed  
  (no different than Manning’s eqn.)
• Trying different equations to evaluate 
  uncertainty by using different transport 
  formulas misses the point:  
  the main source of uncertainty is in the 
  input!

\[
\frac{Q_s}{Q_{sr}} = \left( \frac{Q}{Q_r} \right)^\beta
\]

*If estimating transport rates using shear stress is so hard,  
  remind me again why we can’t find some general formula using Q.*

With an estimate for \( \tau' \), lets return to 
the sediment rating curve problem.  
The goal is to see whether a transport 
relation based on discharge \( Q \) can be 
general.

We approximate transport with a  
power function of \( \tau' \)

\[
q \propto (\tau')^3 \rightarrow q_S \propto \tau'^3
\]

Write the relation twice, take ratio 

\[
\frac{q_s}{q_{sr}} = \left( \frac{\tau'}{\tau_r} \right)^3
\]
\[ \frac{q_s}{q_{sr}} = \left( \frac{\tau'}{\tau_r} \right)^3 \]

We use our grain stress relation:

\[ \frac{\tau'}{\tau_r} = \left( \frac{U}{U_r} \right)^{3/2} \]

Which gives

\[ \frac{q_s}{q_{sr}} = \left( \frac{U}{U_r} \right)^{4.5} \]

For simplicity, we assume the width of transport does not change.

\[ \frac{Q_s}{Q_{sr}} = \left( \frac{U}{U_r} \right)^{4.5} \]

We can relate \( U \) to \( Q \) using the hydraulic geometry:

\[ B = aQ^b \quad \left( \frac{U}{U_r} \right)^{4.5} = \left( \frac{Q}{Q_r} \right)^{4.5m} \]

\[ h = cQ^f \]

\[ U = kQ^m \quad \frac{Q_s}{Q_{sr}} = \left( \frac{Q}{Q_r} \right)^{4.5m} \]

For \( 0.2 < m < 0.8, \ 1.0 < \beta < 3.6 \).

\( \beta \) is not a constant! \( Q \) is not a suitably general variable for a transport model.

Even larger values of \( \beta \) found in steep mountain streams (typically very small transport rates, approximated by \( \beta > 3 \) in transport formula).

We find the exponent to vary between 1 and 10 (!!)
Estimating Bed-Material Transport in Gravel-Bed Rivers

For real ….
How many sizes?
Sample?

Estimating Sediment Transport: Three overarching constraints bound the problem

• Large spatial & temporal variability
  Need BIG samples

• Strongly nonlinear processes
  bed-material transport is a local affair
  Modeling with spatial/temporal averages produces large error

• Sparse information available relative to that needed to model the transport
  Modeling with high spatial/temporal resolution requires vast amounts of data

• Generally, we need a robust approach
Requires decision regarding grains to exclude from the transportable population bedimmobile at typical high flows

Bed Material –

cobble

bed load

interstices, stripes and dunes, subsurface

grain path in near-bed region dominated by larger grains; hard to sample & model

Bed Material –

med-crs gravel, cobble

bed load

bed framework

displacements generally rare and hard to capture

We focus on fine and coarse bed material

But there are more reasons to like a two-fraction transport model!

Robust!

Mappable!

Captures sand/gravel interaction!
What about more fractions?

Many-fraction models available: including ones based on the grain size of the bed surface, which allow for the prediction of transient conditions.

These models are fragile – output is sensitive to the quality of the input.

To simulate transport at a particular location, at a particular time,

need large amounts of detailed bed and flow data

Use these models to ask more general questions:

e.g. change in bed composition & transport with a change in flow releases from dams or a change in sediment supply from dam removal

We will apply such a model later …

Estimating Transport: Sampling

Option 1: trap all the transport in a weir, slot, or pool

Option 2: point samples: portable or pit/net-frame samplers installed on the bed.

Option 1 is best, but generally not practical

Option 2 is ≈ practical, but involves larger error, some risk, some luck, and lots of effort
Really Big River Sampling 2007

Sampling I

Transport field highly variable in space & time

→ Need LARGE samples!

Define *Sampling Intensity* for a 6.5m stream

<table>
<thead>
<tr>
<th>Total Transport</th>
<th>[6.5 \text{m}] \times [3600 \text{s}] = 23,400 \text{ m•s}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hand-held Sampling Transect</td>
<td>[13 \times 0.075 \text{m}] \times [120 \text{s}] = 117 \text{ m•s} (0.5%)</td>
</tr>
<tr>
<td>Pit/Trap Sampler</td>
<td>[5 \times 0.305 \text{ m}] \times [3600 \text{s}] = 5,490 \text{ m•s} (23.5%)</td>
</tr>
</tbody>
</table>

*So, pit or net-frame traps look pretty good …*
Pit samplers fill up at high transport rates

Photo J Pizzuto, U Delaware, home of big samples

So do net-frame samplers

**Estimating Sediment Transport**

- **Direct sampling**
  - + gives a direct relation between Q and Qs
  - - requires a big effort
  - - cannot predict future conditions
  - - error: is it random or systematic?
  - - **Handheld samplers:** Scooping, perching, limited grain size & TINY SAMPLES
  - - **Pit/Net-frame samples:** better sampling, only at low rates

- **Formula predictions**
  - + can predict future changes
  - - highly inaccurate
    - flow hard to scale
    - boundary conditions poorly known
The alternative? Join forces. Need for both accuracy and efficiency indicate that the future is a combination of **simple robust models** and **efficient measurement**.

A first cut, using today’s technology

- If pit/trap samplers can collect good samples of small transport rates, why not use these to calibrate a model of coarse bed-material transport
- **GOOD** samples! (long duration, spatially extensive)
- Two sizes: sand and gravel
- Combine in robust framework that is insensitive to major sources of error

Transport Formula (Gravel)

\[
W_i^* = \begin{cases} 
11.2 \left( 1 - 0.846 \frac{\tau_r}{\tau} \right)^{4.5} & \tau > \tau_r \\
0.0025 \left( \frac{\tau}{\tau_r} \right)^{14.2} & \tau < \tau_r
\end{cases}
\]

Dimensionless transport rate

\[
W^* = \frac{q^*}{\tau_*^{3/2}} = \frac{(s - 1)gq_b}{(\tau / \rho)^{3/2}}
\]

Choice of formula does not make that much difference!

*Formula provides the trend, but the samples provide the accuracy*
Transport formula based on shear stress $\tau$, so we need a drag partition relation to get grain stress from discharge. We return to our Manning-Strickler formula:

$$\tau = 17(SD_{65})^{1/4}U^{3/2}$$

- For Cub River:
- Slope $S = 0.02$
- $D_{65} = 90$ mm
- Mean velocity $U = 0.46 \ Q^{0.42}$
Transport samples provide accuracy by

(1) Establishing grain size $D$ of the transport &

(2) Scaling the flow

Calculate grain stress: $\tau = 17(SD_{65})^{0.25} U^{1.5}$

for two different flows & take the ratio: $\frac{\tau_1}{\tau_2} = \left(\frac{U_1}{U_2}\right)^{1.5}$

Suppose $U = aQ^b$

then $\frac{\tau_1}{\tau_2} = \left(\frac{Q_1}{Q_2}\right)^{1.5b}$

$\frac{\tau}{\tau_r} = \left(\frac{Q}{Q_r}\right)^{1.5b}$

If you calibrate, the transport formula is needed only to
give the change in transport with the change in discharge.
Most of the transport occurs at high flows & you base your estimate on samples at low flow?


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**Estimating bed-material transport in gravel-bed rivers**

- **Conceptual basis**
  - fine and coarse bed material
  - (supply of one affects the transport of the other)
- **Sampling**
  - standard needed for minimum sample size
  - fine bed material – Helley-Smith or ?
  - coarse bed material – pit or net frame samplers
  - big rivers – ????
- **Modeling**
  - 2-fraction model captures essential interaction between fine & coarse
    & facilitates integral measure of reach grain size
  - But it can’t do everything (armoring; change in sand or gravel size)
- **Future**
  - combine simple, robust models with efficient monitoring
  - can be done now in wadeable streams, although effort is non-trivial
And now … SRC &/or BAGS

Software and accompanying manual to support estimates of bed-material transport rates in gravel-bed rivers.

Stream Systems Technology Center, (the “Stream Team”), US Forest Service

*BAGS* (Yantao Cui, Stillwater Science): supports variety of transport formulas, allows variety of input, includes calibrated approach

Manual on why & how to estimate transport rates
(Peter Wilcock & John Pitlick)
includes different options depending on purpose & available data

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**Sediment Transport Primer and BAGS User’s Manual**

*Stream Systems Technology Center*

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**TOWARD A PRACTICAL METHOD FOR ESTIMATING SEDIMENT-TRANSPORT RATES IN GRAVEL-BED RIVERS**

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